

P5


25, 28.7, 29, 29.6, 30.4, 31.3, 32.7, 33.4, 34.3, 34.6

a) Mean = $\frac{309}{10} = 30.9 \mu\text{F}$

x	$x - \bar{x}$	$(x - \bar{x})^2$
25	$25 - 30.9 = -5.9$	$(-5.9)^2 = 34.81$
28.7	$28.7 - 30.9 = -2.2$	4.84
29	-1.9	3.61
29.6	-1.3	1.69
30.4	-0.5	0.25
31.3	0.4	0.16
32.7	1.8	3.24
33.4	2.5	6.25
34.3	3.4	11.56
34.6	3.7	13.69
		$\Sigma = 80.1$

c) Variance = $\frac{80.1}{10} = \underline{\underline{8.01}}$

b) Standard deviation = $\sqrt{8.01} = \underline{\underline{2.83}}$



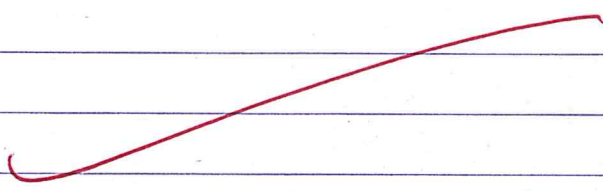
PG

i	f	Midpoint	$M_p \times f$	$M_p - \bar{x}$	$(M_p - \bar{x})^2$	$f(M_p - \bar{x})^2$
20.5 - 20.9	3	20.7	62.1	-1.22	1.4884	4.4652
21.0 - 21.4	10	21.2	212	-0.72	0.5184	5.184
21.5 - 21.9	11	21.7	238.7	-0.22	0.0484	0.5324
22.0 - 22.4	13	22.2	288.6	0.28	0.0784	1.0192
22.5 - 22.9	9	22.7	204.3	0.78	0.6084	5.4756
23.0 - 23.4	2	23.2	46.4	1.28	1.6384	3.2768
	$\Sigma = 48$		$\Sigma = 1052.1$			$\Sigma = 19.9532$

○ a) Mean = $\frac{1052.1}{48} = \underline{\underline{21.92}}$

Variance = $\frac{19.9532}{48} = \underline{\underline{0.416}}$

Standard Deviation = $\sqrt{0.416} = \underline{\underline{0.645}}$



a) Amplitude is the height of the sine wave.
The amplitude in this graph is 1.

$$\omega = \frac{\pi}{2} = 1.57 \text{ rad/s}$$

b) • Periodic time = $\frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4$ seconds.

• Phase angle = 0.6 radians lagging

$$\frac{0.6 \times 180}{\pi} = 34.377^\circ \text{ lagging.}$$

c) • Linear frequency = $\frac{1}{T} = \frac{1}{4}$
= 0.25 Hz

• angular frequency = 1.57 rad/s

P8

$$a) \sin\left(\theta + \frac{\pi}{4}\right) - \sin\left(\theta - \frac{3\pi}{4}\right) = \sqrt{2} (\sin\theta + \cos\theta)$$

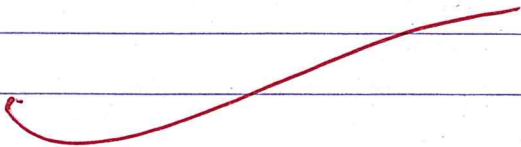
$$\begin{aligned}\sin\left(\theta + \frac{\pi}{4}\right) &= \sin\theta \cos\frac{\pi}{4} + \cos\theta \sin\frac{\pi}{4} \\ &= \sin\theta \times \frac{\sqrt{2}}{2} + \cos\theta \times \frac{\sqrt{2}}{2}\end{aligned}$$

$$\begin{aligned}\sin\left(\theta - \frac{3\pi}{4}\right) &= \sin\theta \cos\frac{3\pi}{4} - \cos\theta \sin\frac{3\pi}{4} \\ &= \sin\theta \times -\frac{\sqrt{2}}{2} - \cos\theta \times \frac{\sqrt{2}}{2}\end{aligned}$$

$$\sin\theta \times \frac{\sqrt{2}}{2} + \cos\theta \times \frac{\sqrt{2}}{2} - \left(\sin\theta \times -\frac{\sqrt{2}}{2} - \cos\theta \times \frac{\sqrt{2}}{2} \right)$$

$$\sin\theta \times \frac{\sqrt{2}}{2} - \sin\theta \times -\frac{\sqrt{2}}{2} = \sin\theta \sqrt{2}$$

$$\cos\theta \times \frac{\sqrt{2}}{2} - \cos\theta \times -\frac{\sqrt{2}}{2} = \cos\theta \sqrt{2}$$

$$\therefore \sin\left(\theta + \frac{\pi}{4}\right) - \sin\left(\theta - \frac{3\pi}{4}\right) = \sqrt{2} (\sin\theta + \cos\theta)$$


$$b) \frac{\cos(270^\circ + \theta)}{\cos(360^\circ - \theta)} = \tan \theta$$

$$\begin{aligned}\cos(270^\circ + \theta) &= \cos 270^\circ \cos \theta - \sin 270^\circ \sin \theta \\ &= 0 \times \cos \theta - (-1) \times \sin \theta \\ &= 1 \times \sin \theta\end{aligned}$$

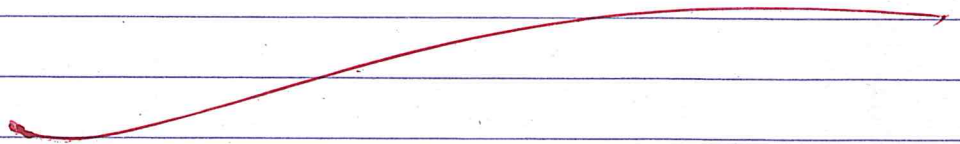
$$\begin{aligned}\cos(360^\circ - \theta) &= \cos 360^\circ \cos \theta + \sin 360^\circ \sin \theta \\ &= 1 \times \cos \theta + 0 \times \sin \theta \\ &= 1 \times \cos \theta\end{aligned}$$

○

$$\frac{1 \times \sin \theta}{1 \times \cos \theta} = \tan \theta$$

$$\therefore \frac{\cos(270^\circ + \theta)}{\cos(360^\circ - \theta)} = \tan \theta$$

○



P8

ii)

$$\cos A = 0.42$$

$$A = \cos^{-1}(0.42) = 65.1654^\circ$$

$$\sin B = 0.73$$

$$B = \sin^{-1}(0.73) = 46.8864^\circ$$

$$\therefore \text{if } A = 65.1654^\circ \quad \text{AND} \quad B = 46.8864^\circ$$

$$\cos A = 0.42$$

$$\cos B = 0.6834$$

$$\sin A = 0.9075$$

$$\sin B = 0.73$$

$$\tan A = 2.1608$$

$$\tan B = 1.0681$$

$$a) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= (0.9075 \times 0.6834) - (0.42 \times 0.73)$$

$$= 0.6202 - 0.3066 = \underline{\underline{0.3136}}$$

$$b) \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= (0.42 \times 0.6834) + (0.9075 \times 0.73)$$

$$= 0.2870 + 0.6625 = \underline{\underline{0.9495}}$$

$$c) \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{2.1608 + 1.0681}{1 - (2.1608 \times 1.0681)} = -2.4678$$

P9

a) $e^t \ln t \cos t$

$$u = \frac{e^t}{u} \frac{\ln t}{\checkmark}$$

$$\dot{u} = ? \quad \dot{v} = -\sin t$$

$$\frac{d}{dt} (e^t \times \ln t) = e^t \frac{d}{dt} (\ln t) + (\ln t) \frac{d}{dt} (e^t) \quad (1)$$

$$\frac{d}{dt} (e^t \times \frac{1}{t}) = e^t \left(\frac{1}{t} \right) \ln t (e^t) \quad (2)$$

$$\frac{d}{dt} (e^t \times \ln t) = e^t \frac{d}{dt} (\ln t) + (\ln t) \frac{d}{dt} (e^t)$$

$$= e^t \left(\frac{1}{t} \right) + \ln t (e^t)$$

$$= e^t \left(-\ln t \sin t + \frac{\cos t}{t} + \cos t \ln t \right)$$

$$= e^t \left(\left(\frac{1}{t} + \ln t \right) \cos t - \ln t \sin t \right)$$

b) $\frac{2xe^{4x}}{\sin^2 x}$

$$\frac{dy}{dx} = \frac{(\sin x)(2x) \times (4e^{4x}) + (e^{4x})(2) - (2xe^{4x})(\cos x)}{\sin^2 x}$$

$$= \frac{8xe^{4x} \sin x + 2e^{4x} \sin x - 2xe^{4x} \cos x}{\sin^2 x}$$

$$= \frac{2e^{4x}}{\sin^2 x} \left[(1 + 4x) \sin x - x \cos x \right]$$

$$c) \frac{1}{(x^3 - 2x + 1)^5} = (x^3 - 2x + 1)^{-5}$$

$$\text{Let } u = g(x) \quad \dot{u} = \dot{g}(x)$$

$$\therefore u = (x^3 - 2x + 1)$$

$$\frac{du}{dx} = \frac{d}{dx}(x^3) - 2 \frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$\circ \frac{du}{dx} = 3x^2 - 2 + 0$$

$$\frac{du}{dx} = \underline{\underline{3x^2 - 2}}$$

$$y = u^{-5}$$

$$\frac{dy}{du} = \frac{d}{du}(u^{-5})$$

○

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{du} = -5u^{-6}$$

$$\frac{dy}{dx} = -5u^{-6} \times (3x^2 - 2) = -5(x^3 - 2x + 1)^{-6} (x^3 - 2x + 1)$$

$$\frac{dy}{dx} = \frac{-5(3x^2 - 2)}{(x^3 - 2x + 1)^6}$$

P10

Definite Integral

$$y = e^{1 - \frac{1}{2}x}$$

$$y = e^1 \times e^{-\frac{1}{2}x}$$

$$y = e e^{-\frac{1}{2}x}$$

$$\int e^{ax} = \frac{1}{a} e^{ax} + c$$

$$\int_0^2 y dx = e \int_0^2 e^{-\frac{1}{2}x} dx$$

$$= e \times \left[\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}} \right]_0^2$$

$$= \frac{e}{-\frac{1}{2}} \left[e^{-\frac{x}{2}} \right]_0^2$$

$$= -2e \left[e^{-\frac{2}{2}} - e^{-\frac{0}{2}} \right]$$

$$= -2e \left[e^{-1} - e^0 \right]$$

$$= -2e \left[e^{-1} - 1 \right]$$

$$= -2e \times \cancel{e^{-1}} + 2e$$

$$= 2e - 2$$

Indefinite Integral

$$a) v = 3 \cos t - 2 \sin t$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt$$

$$\int ds = \int v dt$$

$$s = \int (3 \cos t - 2 \sin t) dt$$

$$s = 3 \int \cos t dt - 2 \int \sin t dt$$

$$\int \cos t dt = \sin t + c$$

$$\int \sin t dt = -\cos t + c$$

$$s = 3 \sin t - 2(-\cos t) + c$$

$$s = 3 \sin t + 2 \cos t + c$$

$$s = 0 \quad t = \frac{1}{2} \pi$$

$$c = -3$$

$$s = 3 \sin t + 2 \cos t + (-3) \text{ m}$$

$$b) t = \frac{1}{2} \pi$$

$$v = 3 \cos \frac{\pi}{2} - 2 \sin \frac{\pi}{2} = -2 \text{ m/s}$$