P5

25, 28.7, 29, 29.6, 30.4, 31.3, 32.7, 33.4, 34.3, 34.6

a) Mean $= \frac{309}{10} = 30.9\mu F$

x	$x - \bar{x}$	$(x-\bar{x})^2$
25	$25 - 30.9 = -5.9$	$(-5.9)^2 = 34.81$
28.7	$28.7 - 30.9 = -2.2$	4.84
29	-1.9	3.61
29.6	-1.3	1.69
30.4	-0.5	0.25
31.3	0.4	0.16
32.7	1.8	3.24
33.4	2.5	6.25
34.3	3.4	11.56
34.6	3.7	13.69
		$\Sigma = 80.1$

c) Variance $= \frac{80.1}{10} = \underline{\underline{8.01}}$

b) Standard deviation $= \sqrt{8.01} = \underline{\underline{2.83}}$

P6

i	f	Midpoint	$Mp \times f$	$Mp - \bar{x}$	$(Mp - \bar{x})^2$	$f(Mp - \bar{x})^2$
20.5 - 20.9	3	20.7	62.1	-1.22	1.4884	4.4652
21.0 - 21.4	10	21.2	212	-0.72	0.5184	5.184
21.5 - 21.9	11	21.7	238.7	-0.22	0.0484	0.5324
22.0 - 22.4	13	22.2	288.6	0.28	0.0784	1.0192
22.5 - 22.9	9	22.7	204.3	0.78	0.6084	5.4756
23.0 - 23.4	2	23.2	46.4	1.28	1.6384	3.2768
	$\Sigma = 48$		$\Sigma = 1052.1$			$\Sigma = 19.9532$

a) Mean $= \frac{1052.1}{48} = \underline{\underline{21.92}}$

Variance $= \frac{19.9532}{48} = \underline{\underline{0.416}}$

Standard Deviation $= \sqrt{0.416} = \underline{\underline{0.645}}$

a) Amplitude is the height of the sine wave.
The amplitude in this graph is 1.

$$\omega = \frac{\pi}{2} = 1.57 \text{ rad/s}$$

b)
- periodic time $= \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4$ seconds.

- Phase angle = 0.6 radians lagging

  $\frac{0.6 \times 180}{\pi} = 34.377^\circ$ lagging.

c)
- linear frequency $= \frac{1}{T} = \frac{1}{4}$

  $= 0.25$ Hz

- angular frequency $= 1.57$ rad/s

P8

a) $\sin\left(\theta + \frac{\pi}{4}\right) - \sin\left(\theta - \frac{3\pi}{4}\right) = \sqrt{2}\left(\sin\theta + \cos\theta\right)$

$$\sin\left(\theta + \frac{\pi}{4}\right) = \sin\theta \cos\frac{\pi}{4} + \cos\theta \sin\frac{\pi}{4}$$
$$= \sin\theta \times \frac{\sqrt{2}}{2} + \cos\theta \times \frac{\sqrt{2}}{2}$$

$$\sin\left(\theta - \frac{3\pi}{4}\right) = \sin\theta \cos\frac{3\pi}{4} - \cos\theta \sin\frac{3\pi}{4}$$
$$= \sin\theta \times -\frac{\sqrt{2}}{2} - \cos\theta \times \frac{\sqrt{2}}{2}$$

$$\sin\theta \times \frac{\sqrt{2}}{2} + \cos\theta \times \frac{\sqrt{2}}{2} - \sin\theta \times -\frac{\sqrt{2}}{2} - \cos\theta \times \frac{\sqrt{2}}{2}$$

$$\sin\theta \times \frac{\sqrt{2}}{2} - \sin\theta - \frac{\sqrt{2}}{2} = \sin\theta\sqrt{2}$$

$$\cos\theta \times \frac{\sqrt{2}}{2} - \cos\theta \times -\frac{\sqrt{2}}{2} = \cos\theta\sqrt{2}$$

$$\therefore \sin\left(\theta + \frac{\pi}{4}\right) - \sin\left(\theta - \frac{3\pi}{4}\right) = \sqrt{2}\left(\sin\theta + \cos\theta\right)$$

b) $$\frac{\cos(270^\circ + \theta)}{\cos(360^\circ - \theta)} = \tan\theta$$

$$\cos(270^\circ + \theta) = \cos 270 \cos\theta - \sin 270 \sin\theta$$
$$= 0 \times \cos\theta - -1 \times \sin\theta$$
$$= 1 \times \sin\theta$$

$$\cos(360^\circ - \theta) = \cos 360^\circ \cos\theta + \sin 360^\circ \sin\theta$$
$$= 1 \times \cos\theta + 0 \times \sin\theta$$
$$= 1 \times \cos\theta$$

$$\frac{1 \times \sin\theta}{1 \times \cos\theta} = \tan\theta$$

$$\therefore \frac{\cos(270^\circ + \theta)}{\cos(360^\circ - \theta)} = \tan\theta$$

P8

ii)

$\cos A = 0.42$

$A = \cos^{-1}(0.42) = 65.1654^\circ$

$\sin B = 0.73$

$B = \sin^{-1}(0.73) = 46.8864^\circ$

$\therefore$ If $A = 65.1654^\circ$ AND $B = 46.8864^\circ$

$\cos A = 0.42$	$\cos B = 0.6834$
$\sin A = 0.9075$	$\sin B = 0.73$
$\tan A = 2.1608$	$\tan B = 1.0681$

a) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= (0.9075 \times 0.6834) - (0.42 \times 0.73)$

$= 0.6202 - 0.3066 = \underline{\underline{0.3136}}$

b) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$= (0.42 \times 0.6834) + (0.9075 \times 0.73)$

$= 0.2870 + 0.6625 = \underline{\underline{0.9495}}$

c) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \dfrac{2.1608 + 1.0681}{1 - (2.1608 \times 1.0681)} = -2.4678$

P9

a) $e^t \ln t \cos t$

$u = \underset{u}{e^t}\ \underset{v}{\ln t}$ $\qquad u' = ? \qquad v' = -\sin t$

$$\frac{d}{dt}(e^t \times \ln t) = e^t \frac{d}{dt}(\ln t) + (\ln t)\frac{d}{dt}(e^t) \quad 1$$

$$\frac{d}{dt}\left(e^t \times \frac{1}{t}\right) = e^t\left(\frac{1}{t}\right) \ln t\,(e^t) \quad 2$$

$$\frac{d}{dt}(e^t \times \ln t) = e^t \frac{d}{dt}(\ln t) + (\ln t)\frac{d}{dt}(e^t)$$

$$= e^t\left(\frac{1}{t}\right) + \ln t\,(e^t)$$

$$= e^t\left(-\ln t \sin t + \frac{\cos t}{t} + \cos t \ln t\right)$$

$$= e^t\left(\left(1\tfrac{1}{t} + \ln t\right)\cos t - \ln t \sin t\right)\right)$$

b) $\dfrac{2xe^{4x}}{\sin x}$

$$\frac{dy}{dx} = \frac{(\sin x)(2x) \times (4e^{4x}) + (e^{4x})(2) - (2xe^{4x})(\cos x)}{\sin x^2}$$

$$= \frac{8xe^{4x}\sin x + 2e^{4x}\sin x - 2xe^{4x}\cos x}{\sin^2 x}$$

$$= \frac{2e^{4x}}{\sin^2 x}\left[(1+4x)\sin x - x\cos x\right]$$

c) $\dfrac{1}{(x^3-2x+1)^5} = (x^3-2x+1)^{-5}$

Let $u = g(x)$ $\quad$ $u' = g'(x)$

$\therefore \quad u = (x^3-2x+1)$

$$\frac{du}{dx} = \frac{d}{dx}(x^3) - 2\frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$\frac{du}{dx} = 3x^2 - 2 + 0$$

$$\frac{du}{dx} = \underline{\underline{3x^2 - 2}}$$

$$y = u^{-5}$$

$$\frac{dy}{du} = \frac{d}{du}(u^{-5})$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{du} = -5u^{-6}$$

$$\frac{dy}{dx} = -5u^{-6} \times (3x^2-2) = -5(x^3-2+1)^{-6}(x^3-2+1)$$

$$\frac{dy}{dx} = \frac{-5(3x^2-2)}{(x^3-2x+1)^6}$$

P10

Definite Integral

$$y = e^{1-\frac{1}{2}x}$$
$$y = e^{1} \times e^{-\frac{1}{2}x}$$
$$y = e\,e^{-\frac{1}{2}x}$$

$$\int e^{ax} = \frac{1}{a}e^{ax} + c$$

$$\int_0^2 y\,dx = e\int_0^2 e^{-\frac{1}{2}x}\,dx$$

$$= e \times \left[\frac{e^{-\frac{1}{2}x}}{-\frac{1}{2}}\right]_0^2$$

$$= \frac{e}{-\frac{1}{2}}\left[e^{\frac{-x}{2}}\right]_0^2$$

$$= -2e\left[e^{\frac{-2}{2}} - e^{\frac{-0}{2}}\right]$$

$$= -2e\left[e^{-1} - e^{0}\right]$$

$$= -2e\left[e^{-1} - 1\right]$$

$$= -2\cancel{e} \times \cancel{e}^{-1} + 2e$$

$$= 2e - 2$$

Indefinite Integral

a) $v = 3\cos t - 2\sin t$

$v = \frac{ds}{dt} \Rightarrow ds = v\,dt$

$\int ds = \int v\,dt$

$s = \int (3\cos t - 2\sin t)\,dt$

$s = 3\int \cos t\,dt - 2\int \sin t\,dt$

$\int \cos t\,dt = \sin t + c$

$\int \sin t\,dt = -\cos t + c$

$s = 3\sin t - 2(-\cos t) + c$

$s = 3\sin t + 2\cos t + c$

$s = 0 \qquad t = \frac{1}{2}\pi s$

$c = -3$

$s = 3\sin t + 2\cos t + (-3)\ m$

b) $t = \frac{1}{2}\pi s$

$v = 3\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2} = -2\ m/s$